A *Treatiſe of* Algebra, *both Hiſtorical and Practical. By* JOHN WALLIS, *D. D. Profeſſor of* Geometry *in the Univerſity of* Oxford; *and a Member of the* Royal Society, London.

THE Author, in his Preface to the Reader, has given us a full account of this learned Work; which ſhall be here Reprinted in his own words.

It contains an Account of the Original, Progreſs, and Advancement of (what we now call) *Algebra*, from time to time; ſhewing its true Antiquity (as far as I have been able to trace it;) and by what Steps it hath attained to the Height at which now it is.

That it was in uſe of old among the *Grecians*, we need not doubt; but ſtudiouſly concealed (by them) as a great Secret.

Examples we have of it in *Euclid*, at leaſt in *Theo*, upon him; who aſcribes the invention of it (amongſt them) to *Plato*.

Other Examples we have of it in *Pappus*, and the effects of it in *Archimedes*, *Apollonius*, and others, though obſcurely covered and diſguiſed.

But we have no profeſſed Treatiſe of it (among them) ancienter than that of *Diophantus*, firſt publiſhed (in *Latin*) by *Xylander*, and ſince (in *Greek* and *Latin*) by *Bachetus*, with divers Additions of his own; and Re-printed lately with ſome Additions of Monſieur *Fermat*.

That it was of ancient uſe alſo among the *Arabs*, we have reaſon to believe, (and perhaps ſooner than amongſt the *Greeks*;) which they are ſuppoſed to have received (not from the *Greeks*, but) from the *Perſians*, and theſe from the *Indians*.

From the *Arabs* (by means of the *Saracens* and *Moors*) it was brought into *Spain*, and thence into *England* (together with the uſe of the Numeral Figures, and other Parts of *Mathematical* Learning, and particularly the *Aſtronomical,*) before *Diophantus* ſeems to have been known amongſt us: And from thoſe we have the name of ALGEBRA.

And indeed moſt of the *Greek* Learning came to us the ſame way; the firſt Tranſlations of *Euclid, Ptolemy*, and others, into *Latin*, being from the *Arabick* Copies, and not from the *Greek* Originals.

The uſe of the *Numeral* Figures (which we now have, but the *Greeks* had not) was a great advantage to the improvement of *Algebra*.

Theſe Figures ſeem to have come in uſe, in theſe Parts, about the Eleventh Century (or rather in the Tenth Century, about the middle of it, if not ſooner;) though ſome others think, not 'till about the middle of the Thirteenth; and it ſeems they did ſcarce come to be of common uſe 'till about that time.

Archimedes (in his *Arenarius*) had laid a good Foundation of ſuch a way of Computation, (as he hath indeed, there and elſewhere, of moſt of thoſe new Improvements, which later Ages have advanced;) Though he has not fitted a Notation thereunto.

The *Sexageſimal* Fractions (introduced, as it ſeems, by *Ptolemy)* did but imperfectly ſupply the want of ſuch a Method of Numeral Figures.

The uſe of theſe Numeral Figures hath received two great Improvements. The one is that of Decimal Parts, which ſeems to have been introduced (ſilently and unobſerved) by *Regiomontanus*, in his Trigonometrical Canons, about the Year 1450; but much advanced in the laſt and preſent Century, by *Simon Stevin*, and Mr. *Briggs*, &c.

And this is much to be preferred before *Ptolemy*'s Sexageſimal way, as is ſhewed by the comparative uſe of both.

And therefore *Briggs*, *Gellibrand*, and others, have attempted the introducing of this, even in thoſe caſes where

where the Sexagesimal is yet in use: Which doth in good measure, now obtain; (and, daily more and more.) And would, no doubt, have obtained absolutely e're this time, did not the Old Tables heretofore Calculated, make it somewhat necessary to retain (in part) the Sexagesimal.

The other Improvement is that of *Logarithms*, which is of great use, especially in *Astronomical* and other *Trigonometrical* Calculations; introduced by the Lord *Neper*, and perfected by Mr. *Briggs* (about the beginning of this Century.) The ground and practice of which is here declared.

And these things, though they be not properly Parts of *Algebra*, are yet of great advantage in the practice of it.

The first Printed Author which Treats of *Algebra* is *Lucas Pacciolus*, or *Lucas de Burgo*, a Minorite Fryer, of whom we have a Treatise in *Italian*, Printed at *Venice* in the Year 1494, (soon after the first Invention of Printing,) and Re-printed there, a while after.

But he therein mentions *Leonardus Pisanus*, and divers others more ancient than himself, from whom he Learned it; but whose Works are not now extant.

This Fryer *Lucas*, in his *Summa Arithmetica & Geometrica*, (for he hath other Works extant) hath a very full Treatise of Arithmetick in all the parts of it; in *Integers*, *Fractions*, *Surds*, *Binomials*; Extraction of *Roots*, *Quadratick*, *Cubick*, &c. and the several Rules of *Proportion*, *Fellowship*, about *Accompts*, *Alligation*, and *False Position*, (so fully, that very little hath been thereunto added to this day:) And (after all this) of *Algebra*, with the Appurtenances thereunto, (as *Surd Roots*, *Negative Quantities*, *Binomials*, *Roots Universal*, the use of the Signs *Plus Minus*, or + —, *&c.*) as far as *Quadratick Equations* reach, but no farther.

And this he tells us was derived from the *Arabs*, (to whom we are beholden for this kind of Learning,) without

 taking

taking notice of *Diophantus* (or any other *Greek* Author) who it seems was not known here in those days.

After him followed *Stiphelius* (a good Author,) and others by him cited, who also proceed no farther than *Quadratick* Equations.

Afterwards *Scipio Ferreus*, *Cardan*, *Tartalea*, and others, proceeded to the Solution of (some) *Cubick* Equations.

And *Bombelli* goes yet farther, and shews how to reduce a *Biquadratick* Equation (by the help of a *Cubick*) to two *Quadraticks*.

And *Nonnius* or *Nunnez* (in *Spanish*;) *Ramus*, *Schonerus*, *Salignacus*, *Clavius*, and others, (in *Latine*,) *Record*, *Digs*, and some others of our own, (in *English*;) did (in the last Century) pursue the same Subject, in different ways; bnt (for the most part) proceeded no farther than *Quadratick* Equations.

In the mean time, *Diophantus*, first by *Xylander* (in *Latin*) and afterwards by *Bachetus* (in *Greek* and *Latin*) was made publick; whose method differs much from that of the *Arabs* (whom those others followed,) and particularly in the order of denominating the Powers; as taking no notice of *Sursolids*, but using only the names of *Square* and *Cube*, with the Compounds of these.

And hitherto no other than the unknown Quantities were wont to be denoted in *Algebra* by particular Notes or Symbols; but, the known Quantities, by the ordinary Numeral Figures.

The next great step, for the improvement of *Algebra*, was that of *Specious Arithmetick*, first introduced by *Vieta* about the Year 1590.

This *Specious Arithmetick*, which gives Notes or *Symbols* (which he calls *Species*) to Quantities both known and unknown, doth (without altering the manner of demonstration, as to the substance,) furnish us with a short and convenient way of Notation; whereby the whole

whole procefs of many Operations is at once expofed to the Ey in a fhort Synopfis.

By help of this he makes many Difcoveries, in the procefs of *Algebra*, not before taken notice of.

He introduceth alfo his Numeral *Exegefis*, of affected Equations, extracting the Roots of thefe in Numbers. Which had before been applied to fingle Equations, fuch as the extracting the Roots of *Squares*, *Cubes*, *&c.* fingly propofed; but had not been applied (or but rarely) to Equations affected.

And in the Denomination of Powers, he follows the order of *Diophantus*; not that derived from the *Arabs*, which others had before ufed.

The method of *Vieta* is followed, and much improved, by Mr. *Oughtred* in his *Clavis* (firft publifhed in the Year 1631.) and other Treatifes of his; and he doth therein, in a brief compendious method, declare in fhort, what had before been the Subject of large Volums: and, doth, in few fmall pieces of his, give us the Subftance and marrow of all (or moft of) the Ancient Geometry.

And for this reafon, I have here inferted a pretty full account of his method, together with an Inftitution for the practice of *Algebra* according thereunto. And, though much of it had been before taught in the Authors above mentioned, yet this I judged the moft proper place to infert fuch an Inftitution, becaufe by him delivered in the moft compendious form.

And in purfuance of his method, and as an Exemplification thereof, I have here added (befide fome Examples of his own) a Difcourfe of *Angular Sections*, and feveral things thereon depending. But this (that it might not feem too great a Digreffion in the body of the Book) I have fubjoined at the end as a Treatife by it felf; as, for the like Reafon, I have done fome other things; to which the principal Treatife doth (in the proper places) refer.

Mr. *Harriot* was contemporary with Mr. *Oughtred*, (but elder than he, and died before him,) and left many good things behind him in writing. Of which there is nothing hitherto made publick, but only his *Algebra* or *Analytice*, which was published by Mr. *Warner*, soon after that of Mr. *Oughtred*, in the same Year 1631.

He alters the way of Notation, used by *Vieta* and *Oughtred*, for another more convenient.

And he hath also made a strange improvement of *Algebra*, by discovering the true construction of *Compound Equations*, & how they may be raised by a Multiplication of *Simple Equations*, and may therefore be resolved into such.

By this means he shews the number of Roots (real or imaginary) in every Equation, and the Ingredients of all the Coefficients, in each degree of Affection.

He shews also how to increase or diminish the *Roots* (yet unknown) by any Excess, or in any Proportion assigned; to destroy some of the intermediate Terms; to turn Negative Roots into Affirmative, or these into those; with many other things very advantagious in the practice of *Algebra*.

And amongst other things, teacheth (thereby) to resolve, not only *Quadraticks*, but all *Cubick* Equations; even those whose Roots have, by others, been thought *Inexplicable*, and but *Imaginary*.

In sum, He hath taught (in a manner) all that which hath since passed for the *Cartesian* method of *Algebra*; there being scarce any thing of (pure) *Algebra* in *Des Cartes*, which was not before in *Harriot*; from whom *Des Cartes* seems to have taken what he hath (that is purely *Algebra*) but without naming him.

But the Application thereof to *Geometry*, or other particular Subjects, (which *Des Cartes* pursues,) is not the business of that Treatise of *Harriot*, (but what he hath handled in other Writings of his, which have not yet the good hap to be made publick;) the design of this being purely *Algebra*, abstract from particular Subjects.

Of

Of this treatise here is the fuller account inserted, because the Book it self hath been but little known abroad; that it may hence appear to what estate *Harriot* had brought *Algebra* before his death.

After this follows an account of Dr. *Pell's* method, who hath a particular way of Notation, by keeping a Register (in the Margin) of the severall Steps in his Demonstrations, with References from one to another.

Of this, some Examples are here inserted of his own, and others in imitation thereof; with intimation how that innumerable Solutions of Undetermined Cases are by his method easily discoverable, where great Mathematicians have thought it a great work to find out some one.

On this occasion there is a farther Discourse of *Undetermined Questions*, and the Limitation of them, and particularly of the Rule of *Alligation*; and of (what they call) *Geometrical Places*; which are of a like nature, and but the *Geometrical* Construction of (some of) these Undetermined Questions.

After this is a Discourse of *Negative Squares*, and the Roots of them; on which depend (what they call) *Imaginary Roots* of Impossible Equations; shewing, what is the true Import thereof in nature, with divers Geometrical Constructions suiting thereunto.

And here also (though by way of Digression, as to the principal Subject) is account given of several Geometrical Constructions, not only of *Quadratick*, but even of *Cubick* and *Biquadratick* Equations.

Then follows a Discourse of the method of *Exhaustions* (used by Ancients and Moderns,) with the foundation of it.

And in pursuance thereof, the *Geometria Indivisibilium* of *Cavalerius*; shewing the true import thereof, and its agreement with the Ancients method of Exhaustions; as being but a compendious Expression thereof,

and

and grounded thereupon ; not any way contrary or repugnant thereunto.

Consequent to this, is the *Arithmetica Infinitorum*, which depends also on the method of Exhaustions; taking that to be Equal, which is proved to differ by less than any assignable Quantity. And a Vindication of the method of Demonstration therein used.

And lastly, the method of infinite *Series*, (as of late they have been called) or continual *Approximations*, (grounded on the same Principles;) arising Principally from *Division*, and Extraction of *Roots* in *Species*, Infinitely continued.

With several Examples of the Application thereof, to the Squaring of Curve-lined Figures, Rectifying of Curve Lines, Planing of Curve Surfaces, and many other perplexed Inquiries.

Which is an *Arithmetick of Infinites upon Infinites*. For when as the *Quotient* of Division, or the *Root* extracted in *Species*, doth not Terminate, but run on Infinitely, (much after the manner of some ordinary Fractions, when reduced to Decimals;) an Infinite Series of these (continued as far as is thought necessary,) is Collected according to the method, in the Arithmetick of Infinites, for Terminated Magnitudes.

This was introduced by Mr. *Isaac Newton*, and hath been pursued by Mr. *Nicholas Mercator*, and others.

And it is of great use for Rectifying of *Curve Lines*, Squaring of *Curve-lined Figures*, and other abstruse Difficulties in *Geometry*; especially where the Enquiry doth not end in a determinate Proportion, explicable according to the commonly received ways of Notation.

And on this occasion, is inserted a Discourse of *Infinite Progressions Geometrical*; (which when decreasing, become Equivalent to Finite Magnitudes,) first used by *Archimedes*, and since pursued by *Torricellius*, *San-Vincent*, *Tacquet*, and others. With the Result of two or more such Progressions compounded.

Several

Several other Discourses are, partly inserted in their proper places, and partly subjoyned at the end, that they might not seem too great Digressions.

And particularly a Treatise of the *Cono-Cuneus* (a Body Compounded of a *Cone*, and a *Wedge*,) with the Sections thereof; considered in the same manner as the Sections of a *Cone* use to be considered.

A Treatise of *Angular Sections*, (a Subject handled by *Vieta*, and others,) with other things thereon depending; together with a short (but full) account of *Trigonometry*.

A further Treatise of the *Angle of Contact*; in pursuance of a former Treatise on that Subject. Wherein is further discoursed what concerns the *Composition of Magnitudes*, *Inceptives of Magnitudes*, *Composition of Motions*, and other things hereunto relating.

A Treatise of *Combinations*, *Alternations*, and *Aliquot Parts*: a Subject discoursed of, by *Schooten*, *Pell*, *Kersey*, and others.

With many other things, which may be seen in the Table of Chapters; but, more fully, in the Treatise it self. Much of which are Additions of my own, where I apprehended a defect (in what I met with in others) which seemed needfull to be supplyed.

But I do not pretend so to have gleaned all those Authors who have Written on this Subject, as to have left nothing worthy to be there sought, in the Authors themselves, (especially as to the Accommodation thereof to particular Subjects:) But have rather directed to those Authors where such things are to be found.

And I have been the less able so to do (if I would have done it,) because I did not designedly read them over to this purpose, nor (when I did read them) did make Collections (as I went along) in order to such a design. But have rather (out of my memory) inserted (in their proper places) such things as the Order and Method of the Difcourfe feemed to call for; and (on fuch occafions) had recourfe to the refpective Authors.

Thofe who defire a fuller account of fuch things as I have but briefly touched; may, for that purpofe, confult *Vieta*, *Oughtred*, *Harriot*, *Cartes*, *Slufius*, and others; and (in Englifh) Mr. *Kerfy*; who hath publifhed a Compleat Volume of *Algebra* (with the Appurtenances thereunto) in Two parts.

But my defign being, to trace this of *Analyticks* (as the *Greeks* call'd it) or *Algebra* (as the *Arabs*) from its firft Original (as near as I could) by the feveral Steps whereby it hath proceeded: Mine Ey was chiefly on the feveral Advances which from time to time it hath made; omitting, for the moft part, the Accommodations thereof to particular Subjects.

And herein I have endeavoured, all along, to be juft to every one: Afcribing, as near as I could, every Step of advance to its own Author; or at leaft to the moft ancient of thofe in whom I found it.

If I have any where miffed of this (afcribing to a latter what was due to fome former Writer;) it is either becaufe I had not read the more ancient, or did not there heed it when I fo read, or at leaft did not remember it, when I was Writing. And I fhall be willing to be rectified, in what I have any where miftaken.

There may yet perhaps (notwithftanding all my care) be fome difficulty to fatisfy all Readers, as to what I have, or what I have not taken notice of. Who may think there are divers things omitted (and doubtlefs there are fo) which might deferve to be taken notice of; or but briefly touched, which might have deferved a fuller difcourfe; and fome things inferted, which (in their opinion) might have been fpared, or needed not to have been fo fully handled.

But as to fuch things, I muft be content to leave my felf to the Readers Candor; or leave the Readers themfelves selves to satisfie one another Amongst whom, some may be found to Blame, what another Commends, and some to Commend, what another Blames.

And I have endeavoured all along to represent the sentiments of others with Candor, and to the best advantage: Not Studiously seeking opportunities of Cavilling, or greedily catching at them if offered. (For there is no man can Write so warily, but that he may sometime give opportunity of Cavilling, to those who seek it.) And have been carefull to put the best Construction on their Words and Meaning; and, if need be (as sometimes there is) to help an incommodious expression, by one (as at least appeared to me) more intelligible and better agreeing (or more fully) to their own meaning; (without reproaching them for the want of such:) For it many times happens, that a man lights on a good notion; which he hath not the happiness to express so intelligibly, as perhaps another may do for him. And if here (sometimes) I have so done (as I think I have;) I do not therein wrong, either the Author or the Reader.

Apologia pro Circuitione Sanguinis; *qua respondetur* Æmylio Parisano, *medico* Veneto, *Authore* GEORGIO ENTIO; *Editio altera, auctior & accuratior* Lond. 1685. 8vo.

THE Learned Author having thought fit to give us a second Edition of this Book, it may not be amiss to take notice of some particulars added in this Impression.

In his Epistle to Dr. *Harvey*, he shews how little truth there is in *Father Paul's* being the Inventor of the *Circulation*; for as much as the papers Written by him on this Subject, and found in his Study after his Death, were no more then notes taken at the reading of Dr. *Harvey's* Book, which was lent him by a Country-man of his lately returned to *Venice* from *England*, where he had been *Ambassador* from that *State*, and was presented by the *Author* with one of these Books; the Truth of which appears from a Letter written by *Father Fulgentio* to Dr. *Harvey*, expressing as much.

In the Book it self he takes occasion to give some *Comparative* account of the *Spleen* in several creatures.

He enquires into the reason why the Air is hotter in Summer then in Winter, which he thinks does not proceed, from that the rays of the *Sun* fall on the Earth, nearer to a perpendicular, in the former then in the latter season; the *Angulus Incidentiæ* made by the Rays not being to be considered so much as the *number* of them; he supposes this effect may more rationally be deduced partly from the Northern Winds blowing much in Winter, but chiefly from the *Density* of the *Air* which defends us from the *Sun*, and prevails more (he thinks) by 40 to one, in the Winter, then in the Summer.

To prove that the Blood does not nourish, he urges the disproportion between some parts, and the quantity of Blood conveyd to them; thus a great deal of Blood is carryed to the Intestines, and but little to the Head, in proportion to the bigness of each part: he urges the largeness of the Veins in respect to the Arterys: and the smalness of the recruit the Blood receives by the *Ductus Thoracicus*, not large enough, the Author thinks, to convey matter for the nourishment of the whole body; with several other particulars to the same effect.

He answers the arguments of Dr. *Needham*, and Dr. *Mayow*, against the *Biolychnium*.

Printed at the Theater *in* Oxford, *for* Sam. Smith *at the Prince's Arms in St. Paul's Church-yard* London; *and* Hen. Clements *Book-seller in* Oxford.